The Institute of Mathematical Sciences
The Chinese University of Hong Kong

IMS Workshop on Partial Differential Equations
February 3, 2012 (Friday) – February 4, 2012 (Saturday)

Venue: Room 501a, 5/F, Academic Building No.1, CUHK

TITLE & ABSTRACT

Non-uniqueness of Leray-Hopf type solutions to the Navier-Stokes equation on a two dimensional negatively curved Riemannian manifold

Chi Hin CHAN
IMS, CUHK

Abstract

In this talk, we will present a piece of joint work with Magdalena Czubak in which we constructed, for a special choice of finite energy initial datum, a family of non-unique Leray-Hopf type solutions to the Navier-Stokes equation on a two dimensional negatively curved manifold. The finite energy initial datum we take arises from the gradient of a non-trivial bounded harmonic function on a general negatively curved manifold. By the term Leray-Hopf type solution associated to a given finite energy initial datum, we understand the one which has finite energy, finite dissipation, and at the same time satisfies the global energy inequality. This non-uniqueness result on a two dimensional negatively curved Riemannian manifold is in sharp contrast with the classical well known uniqueness result for Leray-Hopf solutions to the Navier-Stokes equation on the 2 dimensional Euclidean space. Since this work lies in the borderline of regularity theory of solutions to the Navier-Stokes equations and geometric analysis, necessary background in both areas will be presented.

Unique Continuation on the Analytic Curve and its Applications to Inverse Problems

Jin CHENG
Fudan University & IMS, CUHK

Radial solutions of entire elliptic equations with mixed powers

Juan DAVILA
Universidad de Chile

Abstract

We consider radial solutions of the elliptic equation in the entire space

$$\Delta u + u^p + u^q - \lambda u = 0, \quad u > 0 \qquad (1)$$

where $\lambda > 0$ is a parameter.

We prove, with Manuel del Pino (U. de Chile) and Ignacio Guerra (U. de Santiago de Chile), that for some parameter and some subcritical exponents, there are at least 3 solutions. This result settles in a negative way a question about uniqueness of radial solutions to equations of the form $\Delta u + f(u) - u = 0$.

Nodal soliton solutions for quasilinear Schrodinger equations with critical exponent

Yinbin DENG
Huazhong Normal University & IMS, CUHK

Global well-posedness of the 3D inhomogeneous Navier-Stokes equations

Guilong GUI
IMS, CUHK

Abstract

In this talk, we consider local and global well-posedness of the 3D incompressible inhomogeneous Navier-Stokes equations (INS).

First, we investigate the global well-posedness to (INS) with large initial velocity slowly varying in one space variable in the framework of anisotropic type Besov spaces. And then, without small density variation assumption, we prove the local well-posedness of (INS) with initial data in the critical Besov spaces. Furthermore, if the initial velocity field is small enough in the critical Besov space $\dot{B}^{1/2}_{2,1}$, this system has a unique global solution.

Regularity for a Singular Monge-Ampere equation

Huaiyu JIAN
Tsinghua University

Z-Pinch 內爆等離子體二維三溫輻射磁流體動力學方程

Zhong TAN
Xiamen University & IMS, CUHK

Wetting and contact angle hysteresis

Xiaoping WANG
The Hong Kong University of Science and Technology

Stability of Contact Discontinuities in 3-D Compressible Steady Flow

Yaguang WANG
Shanghai Jiao Tong University & IMS, CUHK

Abstract

In this talk, we study the stability of contact discontinuities in three-dimensional compressible steady Euler equations for supersonic flow. By developing the Kreiss, Majda and Osher's arguments, we obtain the necessary and sufficient weakly stable condition of contact discontinuities.

The energy estimate of solutions to the linearized problem has a loss of regularity.

On Some Properties of Sub-Elliptic Equations

Xiaoping YANG
Nanjing University of Science and Technology & IMS, CUHK

Abstract

Sub-elliptic equations have strong backgrounds in control theory, mathematical physics and other sciences and have been a subject of intensive studies in the last decades. In this talk, we will discuss some properties of sub-elliptic equations and talk about some recent results including asymptotic mean value formulae and growth of sub-harmonic functions.

The Weak Maximum Principle for a Class of Strongly Coupled Elliptic Differential Systems

Xu ZHANG
Chinese Academy of Sciences & Sichuan University

Abstract

A classical counterexample due to E. De Giorgi, shows that the weak maximum principle does not remain true for general linear elliptic differential systems. Since then, there are some efforts to establish the weak maximum principle for special elliptic differential systems, but the existing works are addressing only the cases of weakly coupled systems, or almost-diagonal systems, or even some systems coupling in various lower order terms. In this work, by contrast, we present maximum modulus estimates for weak solutions to some coupled elliptic differential systems with different principal parts, under some mild assumptions. The systems under consideration are strongly coupled in the second order terms and other lower order terms, without restrictions on the size of ratios of the different principal part coefficients, or on the number of equations and space variables.
(This is a joint-work with Dr. Xu Liu)

Some results on semilinear elliptic equations

Huansong ZHOU
Wuhan Institute of Physics and Mathematics & IMS, CUHK

Optimal Decay Rates to Conservation Laws with Diffusion-type Terms

Changjiang ZHU
Central China Normal University

Abstract

We consider the Cauchy problem on nonlinear scalar conservation laws with a diffusion-type source term related to an index $s \in R$ over the whole space R^n for any spatial dimension $n \geq 1$. Here, the diffusion-type source term behaves as the usual diffusion term over the low frequency domain while it admits on the high frequency part a feature of regularity-gain and regularity-loss for $s < 1$ and $s > 1$, respectively. For all $s \in R$, we not only obtain the L^p-L^q time-decay estimates on the linear solution semigroup but also establish the global existence and optimal time-decay rates of small-amplitude classical solutions to the nonlinear Cauchy problem. In the case of regularity-loss, the time-weighted energy method is introduced to overcome the weakly dissipative property of the equation. Moreover, the large-time behavior of solutions asymptotically tending to the heat diffusion waves is also studied. The current results have general applications to several concrete models arising from physics.
(joint with Renjun Duan and Lizhi Ruan)